Sheafhom

A package for sparse linear algebra and algebraic topology

Mark McConnell

Overview

- Package for large-scale mathematical computations
- Front end for algebraic topology and number theory
- Back end solves large sparse systems of linear equations over the integers

History

- Sheafhom 1.*x*, 1993-99, CLtL1/2
- Sheafhom 2.0, 2001-04, Java
- Sheafhom 2.1, 2004-05, ANSI CL
- At http://www.lispwire.com and www.geocities.com/mmcconnell17704

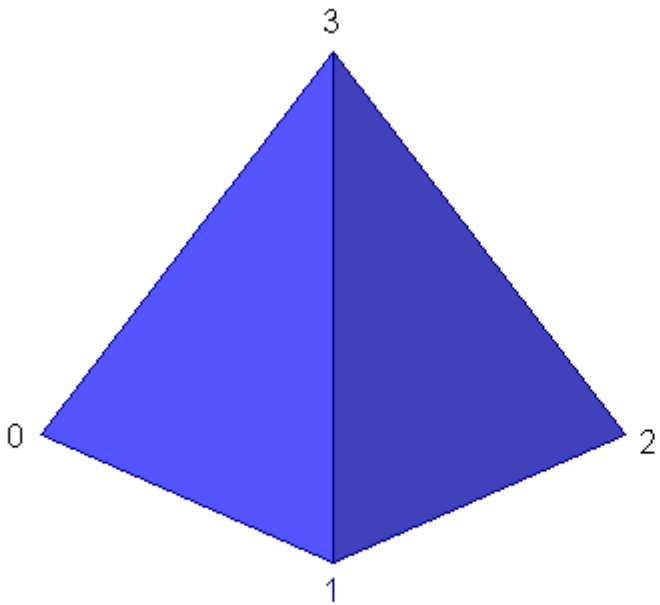
Topology

- Ash, Gunnells, — : cohomology of arithmetic subgroups. Compute Hecke eigenvalues, look for attached Galois representations, as examples of the Langlands program.
- Dimensions 3, 4, 6, embedded in up to 10.
- Today, stick to 2 dimensions.

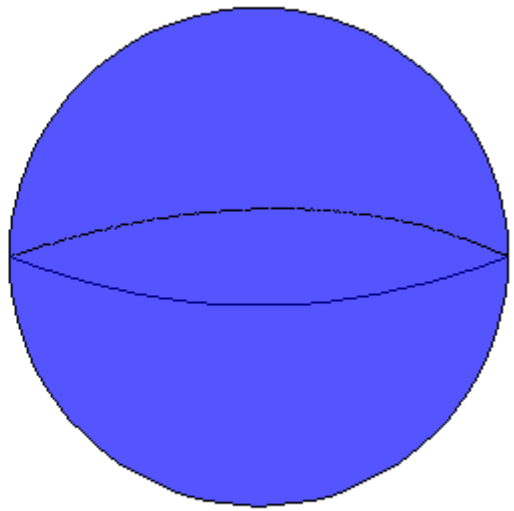
Two-Complexes

- A *two-complex* is a space made by gluing triangles together along their edges and vertices.
- Or any space *homeomorphic* to one of these.

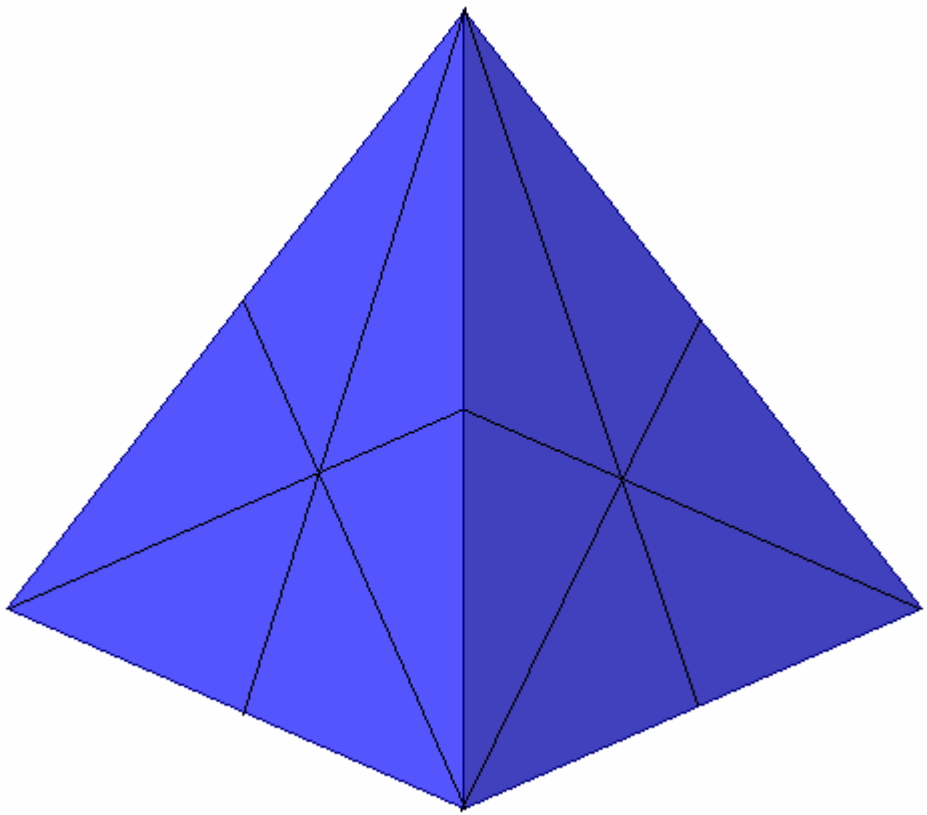
Sphere S^2



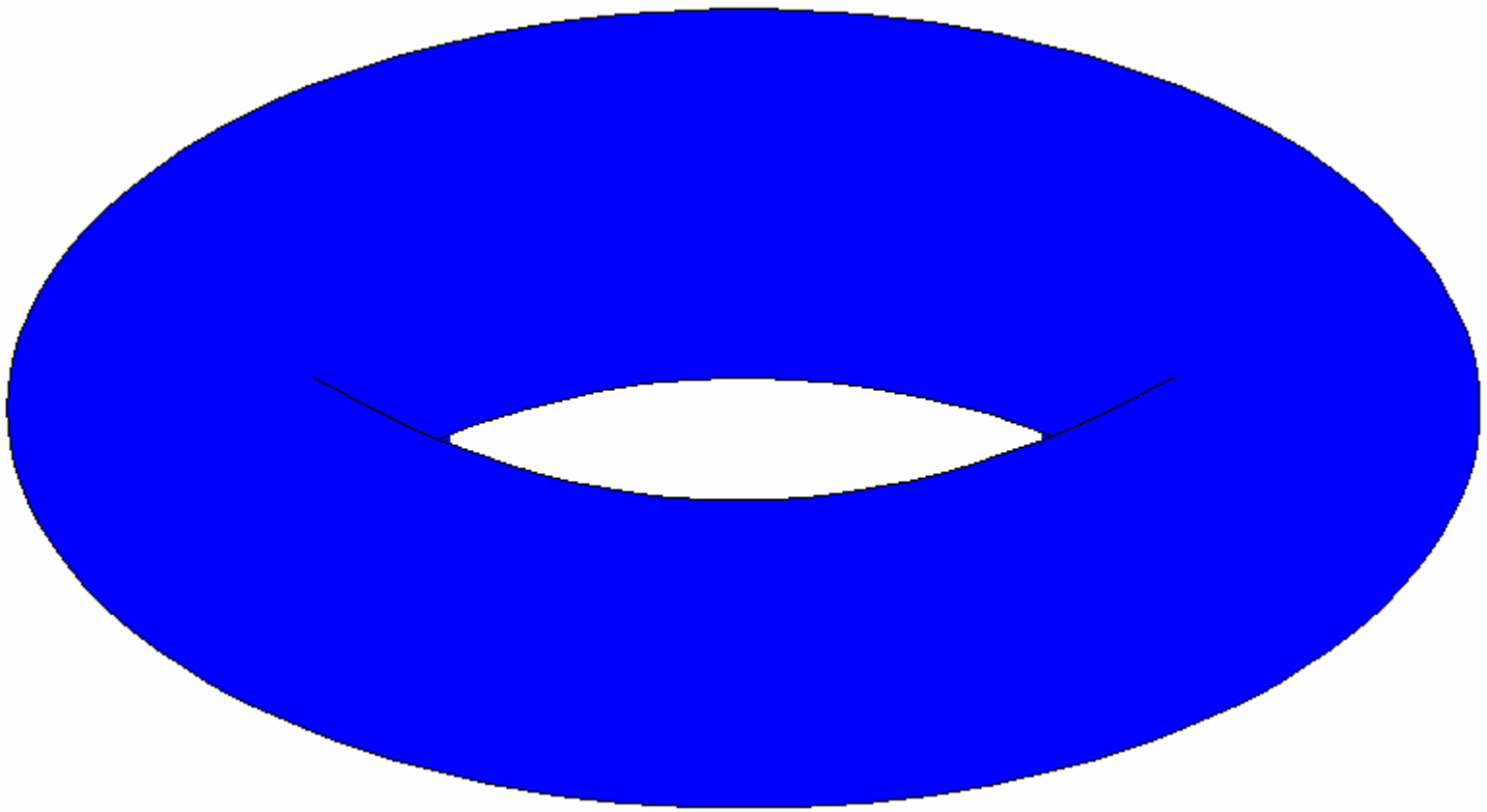
((1 2 3) (0 2 3) (0 1 3) (0 1 2))



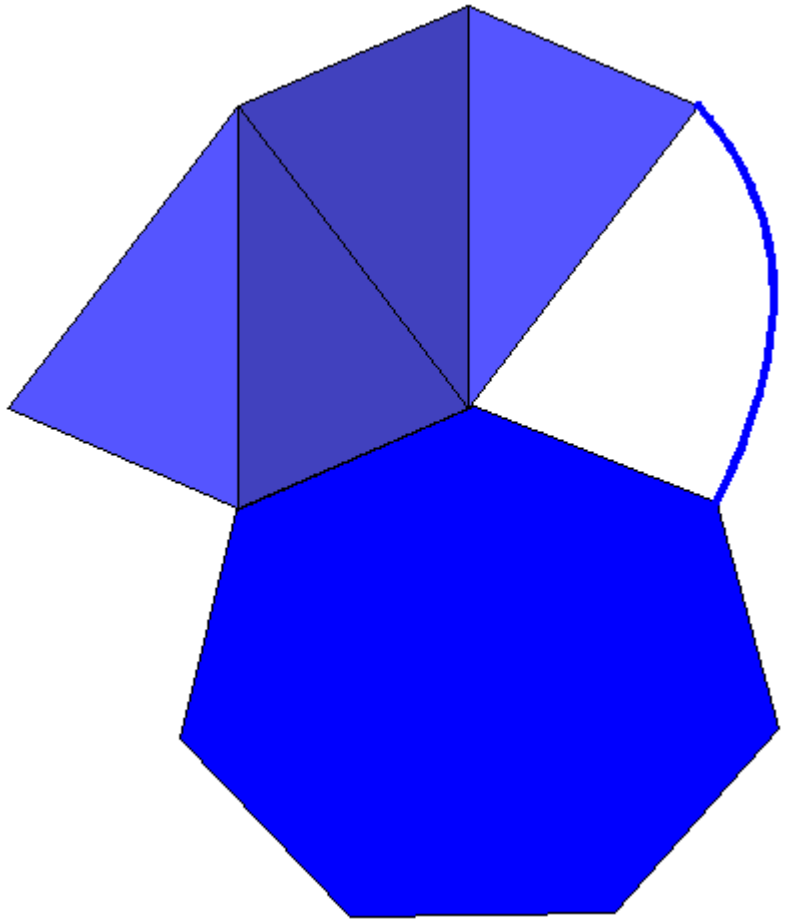
Barycentric Subdivision



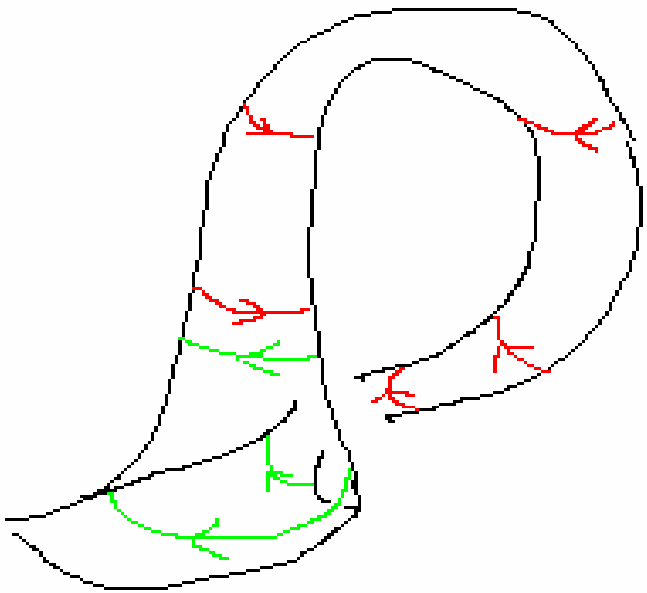
Torus T^2



Others



Klein Bottle (non-orientable)

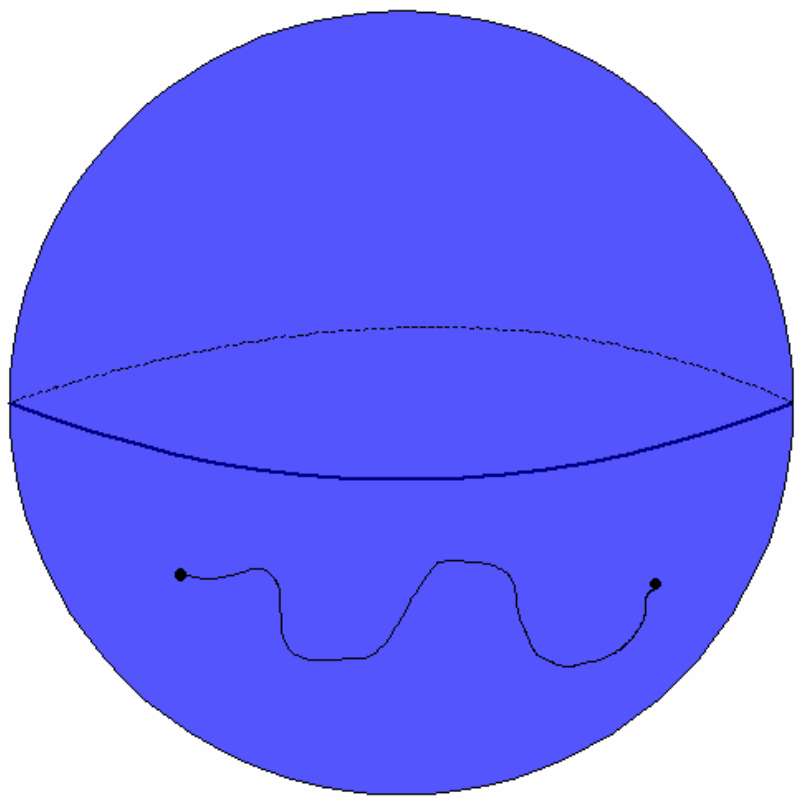


Homology

- Form a group of *i*-dimensional cycles (*i*-dimensional loops) on a space X.
- Two cycles are equivalent if they are the boundary of an (*i*+1)-dimensional object.
- The result is the *i*-th homology $H_i(X)$.
- I'll speak of the rank of H_i, the number of independent generators.

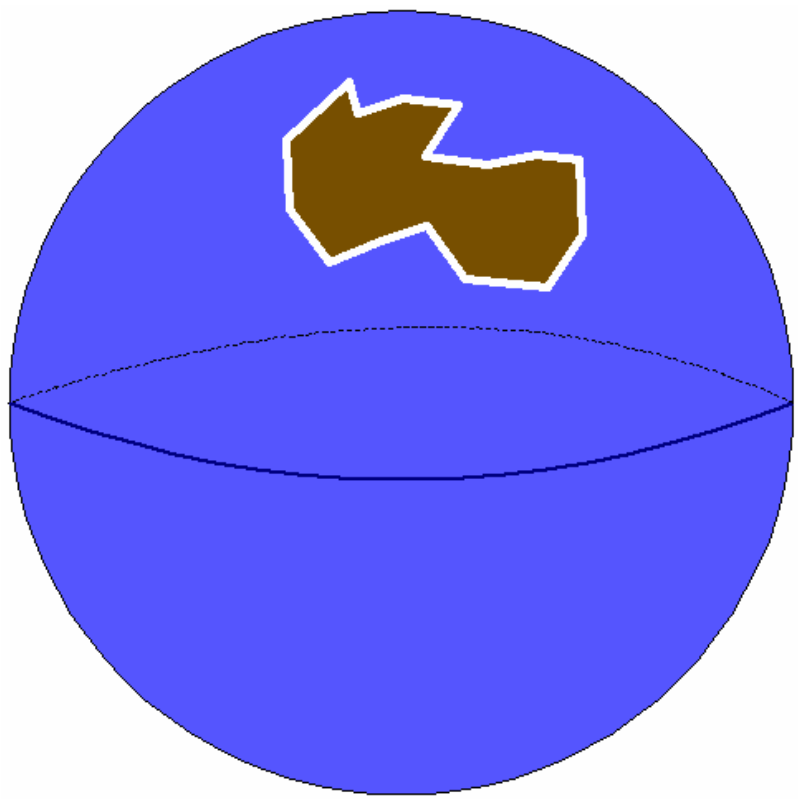
H_0 Measures Connectedness

- H_0 has rank 1



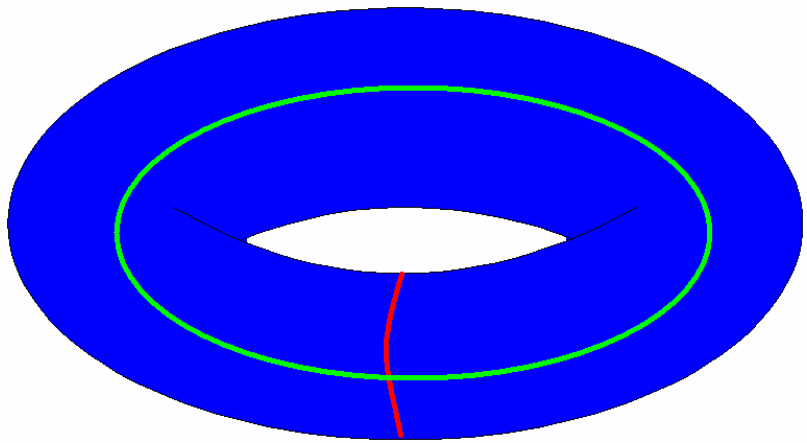
H_1 = Essential Loops

- H_1 has rank 0

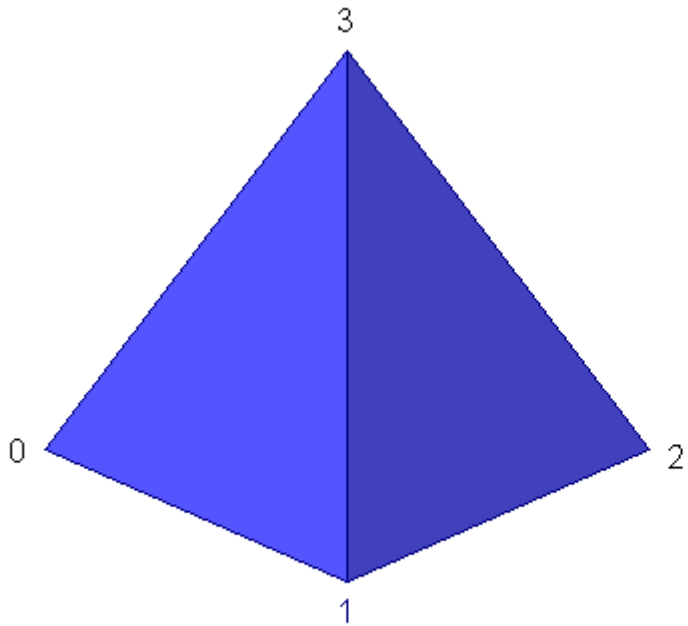


H_1 = Essential Loops

- H_1 has rank 2



Compute Hi With Boundary Matrices



((1 2 3) (0 2 3) (0 1 3) (0 1 2))

$$\begin{matrix} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{matrix}$$

H_1 on non-orientable spaces

- One loop goes all around the outside
- Red loop ≠ green loop
- But 2 × (red loop) = (red + green) = (do something, then undo) = 0.
- *Torsion*: $2 \times x = 0$ yet $x \neq 0$.
- $H_1 = \mathbf{Z} + \mathbf{Z}/2\mathbf{Z}$.